LARGE AMPLITUDE ROLLING IN A REALISTIC SEA

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Abstract

The problem of large amplitude, nonlinear, rolling in the presence of a stochastic beam sea has been approached several times in the past. However, most of the published work is devoted to the consideration of the case where the bandwidth of the excitation (input process) is greater than that of the rolling ship (output process). As a result the complex nonlinear roll dynamics cannot exhibit all its typical peculiarities and in particular no consideration is given to the very dangerous possibility of bifurcations and jumps of amplitude as precursors of an eventual degeneration to chaotic behaviour. This possibility was proposed long ago in the narrow band stochastic case and successively revealed experimentally in regular beam waves. In the meantime the possibility of bifurcations in the presence of stochastic excitation was confirmed together with the validity of Gaussian methods to this purpose. In this paper, the hypothesis of "artificial" narrow band sea spectrum is abandoned and the case of Pierson-Moskowitz case is analysed by means of the cumulant-neglect closure method. The strong effect of non linearities is highlighted together with the possibility of complex roll dynamics.

1. INTRODUCTION

The problem of large amplitude, nonlinear, rolling in the presence of a stochastic beam sea has been approached several times in the past. However, most of the published work was devoted to the consideration of the case where the bandwidth of the excitation (input process) is greater than that of the rolling ship (output process) so that the complex nonlinear roll dynamics cannot exhibit all its typical peculiarities and in particular no consideration is given to the very dangerous possibility of bifurcations and jumps of amplitude as precursors of an eventual degeneration to chaotic behaviour.

One of the authors already developed different stochastic approaches focussing on the nonlinear aspects. The possibility of bifurcations was identified in the presence of narrow band excitation by means of a Gaussian Closure of Moments [1,2] and by means of the Perturbation Method of Multiple Time Scales [3,4]. Later on, the validity of the Gaussian approaches to detect complex dynamics and multiple solution was confirmed by other authors [5]. In both approaches considered in [1-4] the narrow band input was produced by means of a linear filter acting on white noise. A simple linear filter can reproduce any required bandwidth, but generally it distributes too much energy in the low frequency side and thus it cannot properly shape the typical sea spectra or it can give only a rough approximation of them, especially when non sharp peaked functions are considered.

More powerful nonlinear methods for treating stochastic processes have been developed in the meantime. They account for large non Gaussian behaviour by including statistical moments of order higher than that required for Gaussian Closure, although some times simple Gaussian approaches can give surprisingly good fitting of experiments, as observed in [6]. At the same time the simulation of typical sea spectra by means of cascades of linear filters [7-10] was developed and the transfer function and impulse response function for the Pierson Moskowith spectrum were obtained [11].

In this paper we present a novel analysis of large amplitude nonlinear rolling in a stochastic beam sea described by an improved linear filter reproducing known spectra. The cumulant-neglect closure method is employed on different ship nonlinear characteristics to highlight the possibility of complex roll dynamics which in the meantime was experimentally detected in the ship behaviour in regular beam waves [12-13].

2. THE ROLL MOTION MODELLING IN A BEAM WAVES

2.1 Regular waves

The problem of the correct modelling of rolling motion in regular beam waves has been discussed a number of times in the literature on the subject. The conclusions of the different authors are not always in agreement as regards the degree of nonlinearity and the minimum number of degrees of freedom, not to speak of the very basic description adopted, i.e. concentrated parameters or not.

The differences often depend on the specific field of action of the involved persons, stability versus seakeeping or manoeuvrability, regulatory versus research oriented, nonlinear dynamics versus global approaches, etc. On the basis of the previous long experience based on mixed analytical/experimental approaches, in this paper the following assumptions will be made about the mathematical model:

- nonlinear damping and restoring moments;
- frequency dependent excitation;
- one degree of freedom.

The above hypotheses can appear questionable, but in all experiments proved good simulation capability. The first and the third are also the basis for present mathematical modelling of intact stability criteria.
The assumed model will thus be:

$$\ddot{\phi} + 2\mu\dot{\phi} + \delta\dot{\phi}^3 + \omega_0^2\phi + \alpha_3\phi^3 = \pi\omega_0^2 s_w\left(\alpha_0 - \alpha_2\frac{\omega^2}{\omega_0^2}\right)\cos\omega t \qquad (1)$$

A linear plus cubic damping (which can be transformed in a linear plus quadratic one by energy balance), a cubic polynomial righting moment (valid for moderate transversal inclinations) and a quadratic in the frequency excitation to account for diffraction [14] are the main peculiarities of the adopted model. The coefficient α_0 is the effective wave slope coefficient (the highly questioned factor “r” in Weather Criterion), whereas:

$$s_w = \frac{h_w}{\lambda_w} = \frac{2\zeta_a}{\lambda_w} = \frac{\alpha_M}{\pi} \qquad (2)$$

is the wave steepness with α_M the maximum wave slope.

The assumed excitation belongs to the general family containing three terms, respectively related to the wave amplitude or slope and to its first and second derivatives. In the experiments in regular waves it was found that the intermediate term does not play a significant role and it was omitted in present study.

The nonlinear features of Eq. 1 have been thoroughly analysed in the past by means of approximate solutions obtained through perturbation methods [15]. Recently, the existence of bifurcations and jumps of amplitude has been experimentally confirmed in beam waves [12,13].

2.2 Irregular waves

Equation 1 is again the basis for the mathematical modelling of roll motion in a stochastic environment. The excitation, now is irregular and is described by a random process which is generally assumed stationary, ergodic, Gaussian and described by a spectrum. Very often the Pierson-Moskowitz or ITTC and the JONSWAP spectra are assumed for the description of “realistic” sea waves. While these spectral descriptions represent short term approaches and hide particularly dangerous phenomena like wave grouping, on the other hand they are very much used due to their relative simplicity.

The assumed mathematical model for the description of ship rolling in an irregular beam sea is thus:

$$\begin{aligned}\ddot{\phi} + 2\mu\dot{\phi} + \delta\dot{\phi}^3 + \omega_0^2\phi + \alpha_3\phi^3 \\ = \alpha\alpha_M + \alpha'\ddot{\alpha}_M\end{aligned} \tag{3}$$

with:

$$\alpha = \omega_0^2\alpha_0 \quad \text{and} \quad \alpha' \cong \alpha_2 \tag{4}$$

3. THE EXCITATION SPECTRUM

The wave spectrum is usually given in terms of wave amplitude spectrum $S_{\zeta\zeta}(\omega)$. For example the Pierson-Moskowitz (PM) spectrum is given by:

$$S_{\zeta\zeta}(\omega) = \frac{A}{\omega^5}\cdot\exp\left(-\frac{B}{\omega^4}\right) \tag{5}$$

$$\text{with } A = 8.1\cdot10^{-3}\cdot g^2 \text{ and } B = \frac{3.11}{h_{1/3}^2} \tag{6}$$

when expressed in terms of the significant wave height $h_{1/3}$. This spectrum has the traditional unimodal shape shown in Fig. 1 below. The JONSWAP spectrum starts from the previous one considering the effect of a peak enhancement factor γ conventionally ranging in the interval $1 \div 7$ with lower bound corresponding to PM and bandwidth decreasing as γ moves towards the upper bound. When we consider the excitation of angular motions around horizontal axes and in particular rolling, we have to introduce an additional spectrum, which represents the energy spread of waves with reference to the maximum wave slope $S_{\alpha\alpha}(\omega)$:

$$\begin{aligned}S_{\alpha\alpha}(\omega) &= \frac{\omega^4}{g^2}\cdot S_{\zeta\zeta}(\omega) \\ &= \frac{A}{\omega\cdot g^2}\cdot\exp\left(-\frac{B}{\omega^4}\right)\end{aligned} \tag{7}$$

which *is quite different* from previous one, as shown in Fig. 1.

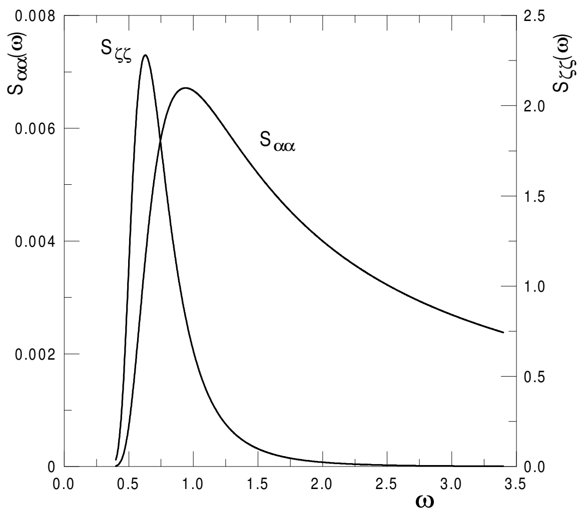

Fig. 1: Comparison between ITTC Spectrum in wave amplitude and in maximum wave slope for $h_{1/3} = 4.0\, m$.

The spectrum reported in terms of maximum wave slope spreads the energy over a much greater frequency interval and moves the peak and a considerable part of the energy towards higher frequencies. These effects are both relevant to the discussion on the effective excitation intensity "perceived" by a low frequency oscillator like the modern passenger cruise ships. Apart this, the bandwidth is considerably increased and the process cannot be in general considered a narrow band one. The bandwidth is in any case limited by the transfer function between the maximum wave slope process $\alpha_M(t)$ and the roll excitation moment process $M(t)$. This transfer function has indeed a natural cut-off frequency when the wave length falls sensibly below ship breadth (say $2 \cdot \lambda_w < B$, being B the ships breadth at waterline).

As a consequence, the excitation can be considered really narrow band only when the basic process is such, as in the case of extreme values of JONSWAP γ factor or in other particular cases like the following waves one. We have not to forget, however, that several times procedures developed for narrow band processes, like for example the Rayleigh distribution of peaks, are applied and work reasonably well in more general cases. The same is often true for other strong hypotheses adopted to allow easy "calculability" like ergodicity, stationarity, Gaussianity, etc.

4. THE REPRESENTATION OF THE SPECTRUM THROUGH A FILTER

For reason which will be more evident later, it is often convenient to represent the random process corresponding to a given spectrum, by means of a linear filter or a cascade of filters. This is also because a filter can represent an idealised spectrum (very close to a real one) of which one can easily change some relevant parameter to adjust the bandwidth, or some other important characteristic, in a controlled way. The use of modern wavemakers allows then to reproduce the situation from an experimental point of view.
Previous approaches to the problem of nonlinear ship rolling in a stochastic beam sea have used:

- a "simple" linear filter and the method of closure of moments at Gaussian level [1,2]:

$$S_1(\omega) \approx \frac{\beta_1^2}{\left[\left(\omega^2 - k_1^2\right)^2 + (c_1\omega)^2\right]} \qquad (8)$$

- a more sophisticated single linear filter and the perturbation method of multiple scales in the hypothesis of narrow bandedness of the filter [3,4,6]:

$$S_2(\omega) \approx \frac{\beta_1^2\omega^2}{\left[\left(\omega^2 - k_1^2\right)^2 + (c_1\omega)^2\right]} \qquad (9)$$

- finally, a cascade of two linear filters has been proposed to solve some problems connected with the motions of offshore structures [7-10]:

$$S_3(\omega) \approx \beta_1^2 \omega^4 / \left[\left(\omega^2 - k_1^2 \right)^2 + \left(c_1 \omega \right)^2 \right] \cdot \left[\left(\omega^2 - k_1^2 \right)^2 + \left(c_1 \omega \right)^2 \right] \tag{10}$$

The capability of these filter approximations to reproduce the wave amplitude and the wave slope spectrum are shown in Fig. 2 and Fig. 3. The parameters defining the filters have been computed by means of a standard least-squares algorithm.

Filter S_1 is reasonable in terms of peak height and bandwidth, but has a non zero value at zero frequency and as such it distributes too much energy in the low frequency range, which usually is a zone where the transfer function to ship roll moment is non negligible. On the other hand, it is not acceptable when used to approximate the maximum wave slope process. The filter S_2 is not so good – not so bad in both cases, but when used to simulate the maximum wave slope process it is not narrowbanded.

Filter S_3 is excellent in all zones. In the following of this paper we shall develop a solution of Eq. 3 for ship rolling in a stochastic beam sea by using filter S_3 to give an analytical approximation of wave spectrum.

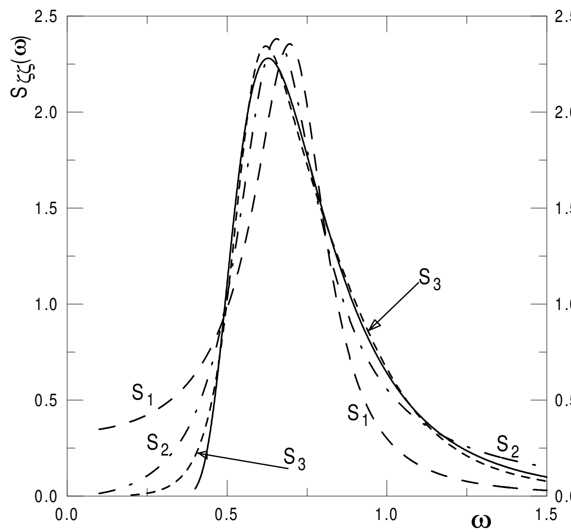

Fig. 2: Comparison between ITTC Spectrum in wave amplitude (solid line) and the various filter approximations for $h_{1/3} = 4.0\, m$.

If filter S_i acts on white noise, it produces a random process whose spectrum is given by the function S_i. Each of them corresponds to a linear differential equation "translating" the effect of the filter transfer function on the white noise process $w(t)$.

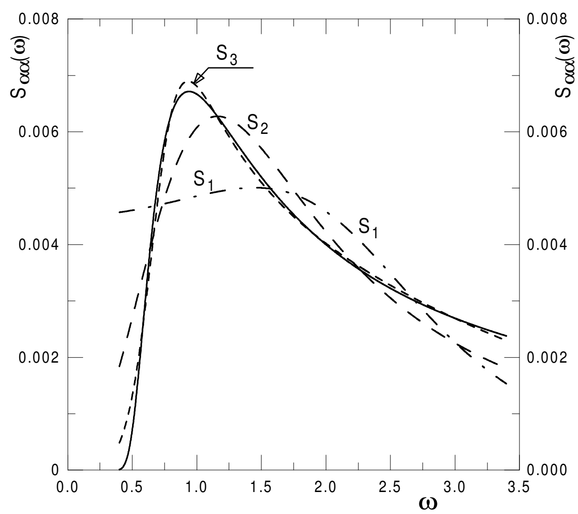

Fig. 3: Comparison between ITTC Spectrum in maximum wave slope (solid line) and the various filter approximations $h_{1/3} = 4.0\, m$.

We have respectively:

$$\ddot{\alpha}_M + c_1\dot{\alpha}_M + k_1^2\alpha_M = \beta_1 w \tag{11}$$

for S_1 ,

$$\ddot{\alpha}_M + c_1\dot{\alpha}_M + k_1^2\alpha_M = \beta_1 \dot{w} \tag{12}$$

for S_2 and finally

$$\begin{aligned}\alpha_M^{IV} + \gamma_1\dddot{\alpha}_M + \gamma_2\ddot{\alpha}_M \\ + \gamma_3\dot{\alpha}_M + \gamma_4\alpha_M = \beta_1\ddot{w}\end{aligned} \tag{13}$$

for S_3, with

$$\gamma_1 = (c_1 + c_2) \qquad \gamma_2 = (k_1^2 + k_2^2 + c_1 c_2)$$
$$\gamma_3 = (c_1 k_2^2 + c_2 k_1^2) \quad \gamma_4 = k_1^2 k_2^2 \tag{14}$$

The simple filter S_1 has a considerable tail at low frequency whereas the other two don't have. This is due to the presence of the frequency in the numerator of the last two. On the other hand, S_2 and S_3 involve derivatives of the white noise process, which do not exist in traditional sense, being white noise not differentiable in any point.

An analytical expression for the transfer function and impulse function transforming white noise in coloured noise following PM spectrum has also been obtained by means of a manipulation of PM formula instead of adopting a linear cascade of filters [11]. This expression can also be used in processes generating the wave trains.

5. THE ROLL MOTION AS A FILTER OF WHITE NOISE

Combining together Eq. 1 and Eq. 14, one has the mathematical modelling of random rolling in a stochastic sea of spectrum given by PM (approximated by S_3). The system can be put in normal form in a way as to avoid the derivatives of the white noise process $w(t)$:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -2\mu x_2 - \delta x_2^3 - \omega_0^2 x_1 - \alpha_3 x_1^3 \\ \qquad + \alpha_{eq} x_3 - \alpha_2\gamma_1 x_4 + x_5 + \beta_1\alpha_2 w(t) \\ \dot{x}_3 = x_4 - \gamma_1 x_3 \\ \dot{x}_4 = x_5 - \gamma_2 x_3 + \beta_1 w(t) \\ \dot{x}_5 = x_6 - \gamma_3 x_3 \\ \dot{x}_6 = -\gamma_4 x_3 \end{cases} \tag{15}$$

having posed:

$$x_1 = \phi \quad x_2 = \dot{\phi} \quad x_3 = \alpha_M \quad \ldots \tag{16}$$

and

$$\alpha_{eq} = \left(\alpha - \gamma_2\alpha_2 + \gamma_1^2\alpha_2\right)$$

With the choice of using the filter to produce coloured noise starting from a white noise process, which is included in the differential equations, the System 15 is Markov with additive Gaussian white noise [16].

We just remind that a random process is Markov when the probability law of the process in future, once it is in a given state, does not depend on how the process arrived at the given state and hence Markov property is a generalised causality principle and, as such, it is a basic assumption that is made in the study of stochastic dynamical systems.

System 15 can be cast in the form of an Ito's differential equation:

$$d\vec{x}(t) = \left[A(t)\cdot\vec{x} + \varepsilon h\left(\vec{x},t\right)\right]dt + CdB(t) \tag{17}$$

where the nxn A matrix of the linear part of System 15 has been separated from the nonlinear n-vector $h(t)$ (the system is supposed to be weakly nonlinear due to the small parameter ε) and acknowledgement was made of the fact that the white noise process $w(t)$ is the derivative (in generalised sense) of a Wiener process $B(t)$ (Brownian motion). C is an nxm constant matrix representing the white noise action on the differential system. The white noise has level S_0 and it is a delta-correlated random process.

It follows from the above that the transition probability density function of the response, $f(x,t|x_0,t_0)$, i.e. the probability density that the system will be in state x at time t given it was in state x_0 at time t_0, must satisfy the Fokker-Planck-Kolmogorov (FPK) equation:

$$\frac{\partial f}{\partial t} = -\sum_{j=1}^{n} \frac{\partial}{\partial x_j}\left[\left(\sum_{k=1}^{n} A_{jk}x_k + \varepsilon h_j\right) f\right] + \sum_{i,j=1}^{n} \frac{\partial^2}{\partial x_i \partial x_j}\left[\left(CDC^T\right)_{ij} f\right] \tag{18}$$

If we now consider the generic polynomial function:

$$g(x) = x_1^{k_1} \cdot x_2^{k_2} \cdot \ldots \cdot x_n^{k_n} \tag{19}$$

with

$$k_1 + k_2 + \ldots + k_n = k$$

the statistical moment of order k can be calculated by evaluating the expectation value of the function g(x), i.e.:

$$E[g(x)] = \int_{-\infty}^{\infty} \ldots\ldots \int_{-\infty}^{\infty} g(x) f(x_0,t_0) f(x,t|x_0,t_0)\,dx\,dx_0 \tag{20}$$

We finally get the following evolutionary equation for the k-th order moment:

$$\frac{d}{dt}E[g] = \sum_{j=1}^{n} E\left[\left(\sum_{k=1}^{n} A_{jk}x_k + \varepsilon h_j\right)\frac{\partial g}{\partial x_j}\right] + \sum_{i,j=1}^{n} E\left[\left(CDC^T\right)_{ij} \frac{\partial^2 g}{\partial x_i \partial x_j}\right] \tag{21}$$

which can be further simplified by remembering the linearity of the expectation operator.

6. THE CLOSURE OF MOMENTS

The procedure given in previous section allows an easy computation of the expressions giving the evolutionary equations of the 6 first order and of the 21 second order moments of System 15. The stationary solution is then described by a set of algebraic equations obtained by setting to zero all the time derivatives. It is immediately evident that the obtained system is not closed since, due to the presence of the non linear terms, the 27 equations for the moments up to second order involve also third and fourth order moments. When additional equations for the higher moments are derived, again by means of Eq. 21, even higher moments are introduced i.e. an infinite hierarchy forms.

To sort out, a way to evaluate the higher order moments in terms of lower order ones has to be introduced and this originates the so called "closure of moments methods". Alternatively, an adjustable non Gaussian probability distribution is constructed which contains a number of undetermined parameters equal to the number of independent equations relating unknown moments [17].

This last procedure makes use of the truncated Gram-Charlier expansion, based on Hermite polynomials to represent the probability density, given the orthogonality relations among the Hermite polynomials and between these and the standardised normal probability density function. As a consequence, the following truncated series representation can be introduced:

$$p(x) = \frac{\exp\left[-\frac{(x-m)^2}{2\sigma^2}\right]}{\sqrt{2\pi}\sigma} \left[1 + \sum_{n=3}^{N} \frac{c_n}{n!} H_n\left(\frac{x-m}{\sigma}\right)\right] \quad (22)$$

where m and s are mean value and standard deviation of the random variable x, while $H_n(\xi)$ is the n-th Hermite polynomial. The probability density defined by Eq. 22 has mean value m and variance σ^2 independently on the order N and on the set of constants $c_1,...,c_n$. These constants are linear combinations of the normalised central moments and Eq. 22 reduces to a Gaussian distribution when all the c_i vanish, hence it is particularly useful to approximate non-Gaussian distributions.

The other approach originates the so called "Closure of moments" methods.

In this paper we remain to second order. The simplest way to do that would be just to neglect the moments of 3rd and 4th order, but this is equivalent to neglect at all the effect of nonlinearities. As a consequence, it can only be used to obtain a first guess solution for a better approach. This is based on neglecting the cumulants of 4th order and higher; this allows higher order moments to be expressed by means of lower order ones as:

$$m_{ijkl} = E\lfloor x_i x_j x_k x_l \rfloor = m_{ij} m_{kl} + m_{ik} m_{jl} + m_{il} m_{jk} \quad (23)$$

and the effect of the nonlinearities is explicitly taken into account although in an approximate way. This method is thus called Gaussian closure of moments or 4th order cumulant neglect, because due to the closure, all the moments can be expressed by means of first and second. Gaussian closure of moments, on the other hand, is equivalent to statistical linearisation and it is widely used (also beyond its validity).

The stationary first order and third order moments all vanish and we obtain a nonlinear algebraic system of 21 equations for the stationary second order moments (actually the system order reduces a bit due to the structure of the dynamical system). This is solved in linear approximation, by setting to zero all coefficients of nonlinear terms deriving from vector h. This linear solution is then used as first guess for a multidimensional Newton-Raphson nonlinear system solver. This last is not very efficient, but it was implemented just for a first look to the results. Better solution will be implemented in next step.

Preliminary results, shown in Fig. 4 to Fig. 6, highlight the important effect of nonlinearities in the case of realistic wave spectrum.

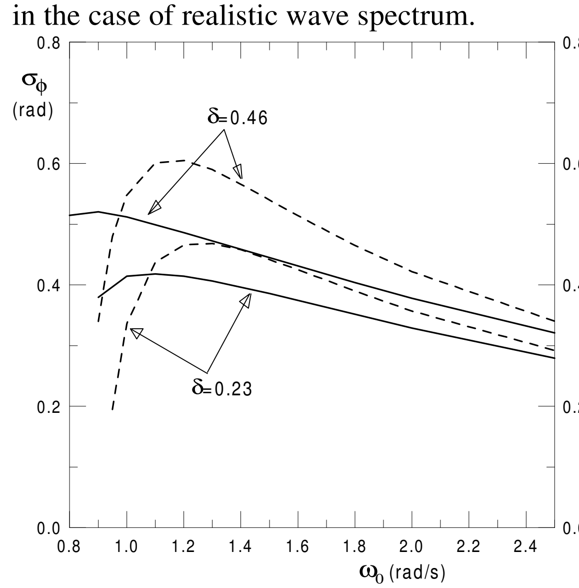

Fig. 4: Effect of righting arm non linearity and nonlinear damping on the roll rms value in the case $\alpha_3 > 0$ (stiffness more than linear). Two different values of nonlinear damping have been used. The dashed curves refer to the linear restoring moment case.

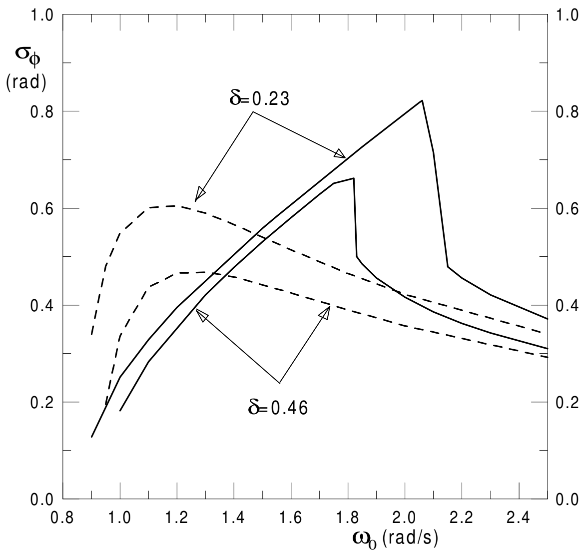

Fig. 5: Effect of righting arm non linearity and nonlinear damping on the roll rms value in the case $\alpha_3 < 0$ (stiffness less than linear). Two different values of nonlinear damping have been used. The dashed curves refer to the linear restoring moment case.

First of all, it could appear there is a wrong indication of the sign of the righting arm non linearity. The behaviour of Fig. 3 and Fig. 4, indeed is typically reported in studies on roll motion as cases with $\alpha_3 > 0$. The explanation is simple, being usually the response amplitude diagram given with respect to the tuning factor ω / ω_0 and assuming that ω is the real "variable".

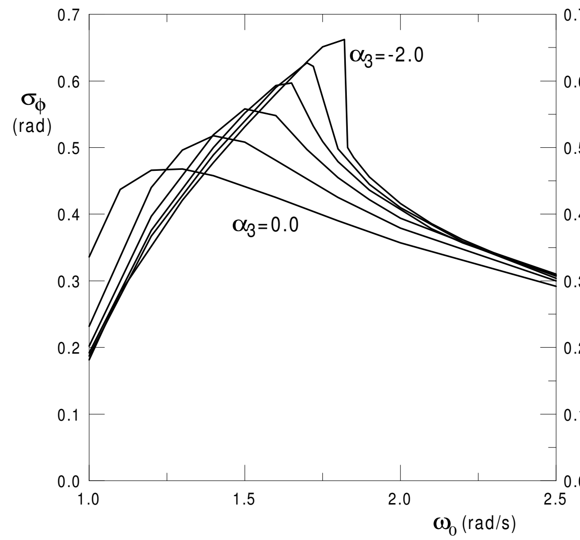

Fig. 6: Effect of righting arm non linearity on the roll rms value in the case $\alpha_3 < 0$ (stiffness less than linear). Intermediate values of $\alpha_{3=}$ 1.5, 1.75, 1.875, nonlinear damping $\delta = 0.46$.

Here we are using a real typical sea spectrum where the peak frequency is a bit changing with significant wave height, but that's all. Once considered a significant wave height, the significant frequency for energy transfer is fixed. Actually, this is also a quite spread interval instead than a single point when we consider the maximum wave slope spectrum. To try to highlight the nonlinear features and the possible nonlinear dynamics peculiarities of the ship at sea, here the natural wave frequency of the ship was varied, or alternatively, different but similar ships have been considered to be subject to the same wave train. Of course, varying the natural frequency would entail changes in the other relevant parameters too, but this was neglected in this first study.

A comparison between linear in the restoring and nonlinear in the restoring is quite interesting, especially in the case $\alpha_3 < 0$, which is the most common for medium size ships. It appears that neglecting the righting arm non linearity could lead to a significant underestimate of the rms rolling. It is also interesting to note that the high bending of the response curve in Fig. 4 seems to conduct to the possibility of complex dynamics with multiple values. The actual possibility of this phenomenon will be clarified in further studies by improving the solution of the nonlinear system of algebraic equations and by going to the non-Gaussian approach either following the cumulant neglect or the Gram-Charlier way.

The cumulant neglect method tends to become not so straightforward by increasing the order, since it leads to a large number of equations. There is however some possibility to make all the process through an automated procedure, as indicated in [18], where also some limits of the method (convergence) have been indicated. The non-Gaussian aspects are non negligible, as the numerical simulations have shown. In particular the kurtosis can be quite far from the Gaussian value. We have not to forget, however, that these non-Gaussian effects are expected to play a major role in the long term prediction, while the spectrum in itself has been conceived as a short term description.

7. CONCLUSIONS

A nonlinear study of ship rolling in a stochastic beam sea represented by the Pierson-Moskowitz spectrum has been conducted. The Gaussian method of the cumulants neglect has been used to obtain an approximated solution for the variance of the roll random process. Preliminary results indicate that the nonlinear terms play a relevant role and that it could be possible that complex nonlinear dynamics takes place in particular cases.

8. ACKNOWLEDGMENTS

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